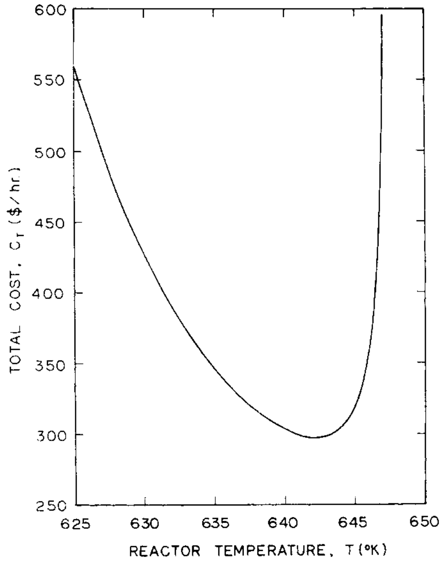

Fig. 3. Total cost versus reactor temperature with T_H = 650°K, T_0 = 645°K, x = 0.000285 gm.-mole/cu. cm., R = 1.9872 cal/gm.-mole °K. Rest of parameters same as in Case 1 of Gaitonde and Douglas (1).

corresponds to the dashed curve in Figure 1 in this communication, we observe a minimum. This minimum in the curve is caused by the choice of T_H and T_0. Since the inlet temperature of the heating fluid T_H does not have to be constrained at 606°K, we may perform a run in which we allow this constraint to be more relaxed. Figure 3 shows the result of such a run with $T_H = 650$, $T_0 = 645$, and $x = 0.000285$. The total cost C_T is reduced from 785 to 297; and the "operating cost" is reduced from 274 to 185. We can therefore increase the yield from 71.5% to 97.2% and also reduce the overall cost substantially by allowing the inlet temperature of the heating fluid to be increased.

Another important consideration is the means of operation and control. Using the same example, we realize at once that we have considerable freedom for choosing the means of supplying heat to the reactor and controlling the system. We are not limited to the use of a heating fluid; nor do we have to apply heat only to the reactor, for we may very well preheat the inlet stream.

These few fundamental considerations must be carefully examined before deciding how to operate and control the system. It is thus premature to state that by allowing the reactor to oscillate one obtains "a 5% lower operating cost than the optimum steady state plant" as is claimed by Heberling et al. (2).

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Heat Transfer through a Wavy Film on a Vertical Surface

W. K. LEONARD and J. ESTRIN

Chemical Engineering Department
Clarkson College of Technology, Potsdam, New York 13676

Heat and mass transfer into and through liquid films occurs frequently in industrial process equipment. In most instances wave motion is encountered at the liquid-gas or liquid-vapor interface, and its presence probably explains why experimental measurements differ from theoretical predictions based on smooth laminar film flow. Experimental measurements show that wave motion intensifies heat and mass transport. This work is a computational investigation of heat transfer through wavy films.

The method by which vertical condensers are designed considers the resistance to heat transfer due to the condensate film which frequently is in transition flow. That the condensate is not in smooth film flow is treated by assuming the existence of turbulence and describing it by established eddy diffusivity methods (3). These design methods permit the analysis of heat transfer through the film for all condensate Reynolds numbers. An alternative description of the flow in the frequently encountered transition region would be laminar wavy flow.

Kapitza (7) presented an approximate analysis of heat transfer in periodic flows noting the effects that increase heat transfer. These are the decrease in the effective average film thickness due to the wave shape, an average film thickness less than the smooth film thickness for the same flow rate, and the convective effects of the periodic flow. Using the values of wave amplitude and film thinning from his film flow model, Kapitza estimated wavy film heat transfer to be 21% greater than that for the equivalent smooth film.

A generalized fluid mechanical model, based on the Kapitza model, has been assumed to describe heat transfer in wavy laminar film flows. The terminology is provided by Figure 1. Steady periodic wave motion is assumed with the surface shape described as a fluctuation about a constant value h_0

$$h = h_0(1 + \phi) \tag{1}$$

where ϕ is a function of $(x - ct)$. The x-component of

Correspondence concerning this communication should be addressed to J. Estrin. W. K. Leonard is with 3M Company, St. Paul, Minnesota.

velocity u is assumed to be given by the parabolic smooth film profile with the average velocity and film thickness replaced by fluctuating quantities

$$u = 3\overline{u}(\phi)\left[\frac{y}{h(\phi)} - \frac{y^2}{2h(\phi)^2}\right] + \frac{3h(\phi)\tau_s}{4\mu}\left[\frac{y^2}{h(\phi)^2} - \frac{2y}{3h(\phi)}\right] \quad (2)$$

where a constant shear stress at the free surface is included. From mass balance considerations the average velocity is given by

$$\overline{u}(\phi) = \frac{u_0 + c\phi}{1 + \phi} \quad (3)$$

and continuity gives

$$v = -\frac{\partial}{\partial x}\int_0^y u\, dy \quad (4)$$

The wave properties are kept arbitrary up to this point: they may be chosen to correspond to any of the Kapitza-like model predictions, or experimental values may be used. They are left arbitrary here so that the effects of their variations on transport through the film may be studied.

The problem of heat transfer through a laminar falling film with constant wall and free surface temperatures for all $x > 0$, is considered here. Since the wave motion is assumed periodic, the following transformation of coordinates is useful:

$$\dot{x} = x - ct, \quad y = y \quad (5)$$

This new coordinate system moves parallel to the wall with a velocity equal to the wave celerity c. Beyond some length at which entrance effects have diminished, that is, at some appropriately large x, the process is described as follows:

$$\frac{\rho C_p}{k}\left[\dot{u}\frac{\partial T}{\partial \dot{x}} + v\frac{\partial T}{\partial y}\right] = \frac{\partial^2 T}{\partial y^2} \quad (6)$$

$$T(\dot{x}, 0) = Tw$$

$$T(\dot{x}, h) = Ts$$

$$T(\dot{x}, y) = T(\dot{x} + i, y), \quad i = 1, 2, 3, \ldots . \quad (7)$$

where

$$\dot{u} = u - c$$

In the $(\dot{x}, y)$ coordinate system this process is both established and steady. Axial conduction has been neglected. The justification for this is discussed elsewhere in depth (4).

In terms of dimensionless variables the energy equation becomes

$$N_{Pe}\frac{h_0}{4\lambda}\left[-U\left(\frac{\phi'}{1+\phi}Y\frac{\partial\theta}{\partial Y} + \frac{\partial\theta}{\partial X}\right) + V\frac{1}{1+\phi}\frac{\partial\theta}{\partial Y}\right] = \frac{1}{(1+\phi)^2}\frac{\partial^2\theta}{\partial Y^2} \quad (8)$$

where

$$\theta = \frac{T - T_w}{T_s - T_w}, \quad X = \frac{\dot{x}}{\lambda}, \quad Y = \frac{y}{h} \quad (9)$$

$$U = \frac{\dot{u}}{u_0}, \quad V = \frac{\lambda v}{h_0 u_0}, \quad \phi' = \frac{d\phi}{dx} \quad (10)$$

and the boundary conditions are

$$\theta(X, 0) = 0, \quad \theta(X, 1) = 1, \quad \theta(X + i, Y) = \theta(X, Y); \quad i = 1, 2, 3, \ldots . \quad (11)$$

The surface shape $\phi(X)$ has been left arbitrary but there are some limitations—the function $\phi(X)$ must be periodic, that is, $\phi(X) = \phi(X + i)$ and must possess a finite first derivative $\phi'(X)$. The pertinent parameters for a specified wave form are the dimensionless wave celerity $Z = \frac{c}{u_0}$, surface shear $\tau_s^* = \frac{h_0\tau_s}{\mu u_0}$, wave amplitude α, and the

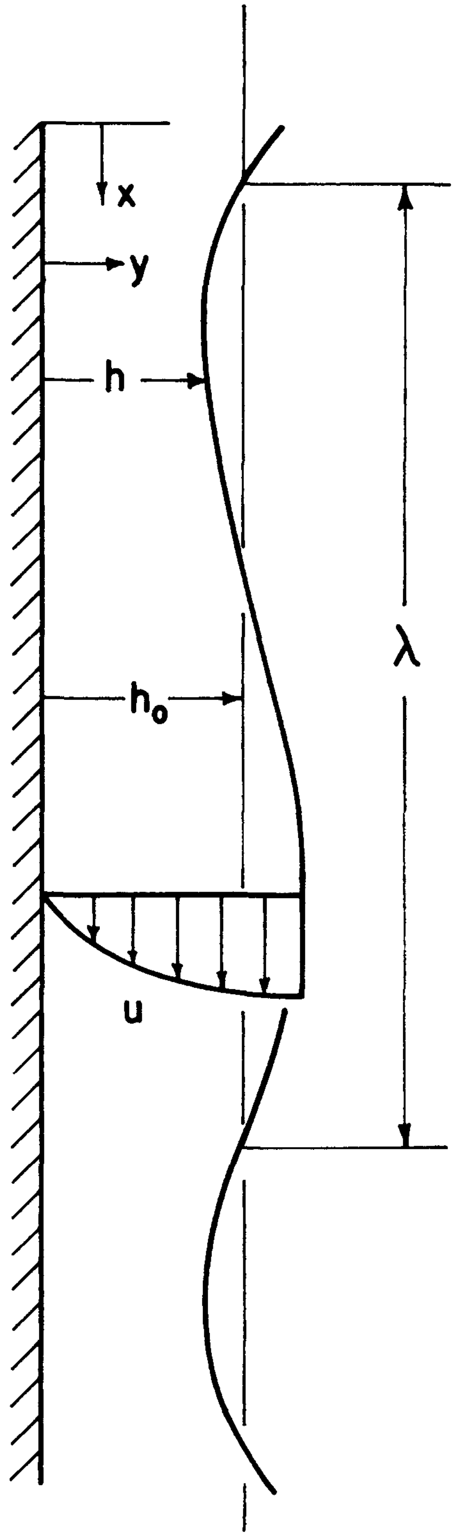

Fig. 1. Diagram of the physical model.

group $N_{Pe} \frac{h_0}{4\lambda}$. The h_0/λ ratio appears as a separate parameter only when axial conduction is considered.

Because of the complexity of the convection terms in Equation (8) a numerical solution was obtained by employing a weighted residual integral technique, the method of moments. The distribution of θ across the film was assumed to be given by a polynomial which may be arranged to give the following form

$$\theta = \left[1 - \sum_{n=1}^{m-1} A_n(X)\right] Y^m + \sum_{n=1}^{m-1} A_n(X)\, Y^n \quad (12)$$

This satisfies the Y-boundary conditions and contains as a special case the smooth film fully developed profile: $\theta = Y$. The above expression for θ was substituted into the residual R formed by subtracting the left-hand side of Equation (8) from the right-hand side and solving the moment equations

$$\int_0^1 RY^n\, dY = 0, \quad n = 0, 1, 2, \ldots m-2 \quad (13)$$

for the unknown functions $A_n(X)$. These equations are linear, first-order coupled differential equations for the A_n's, and the periodicity boundary condition must be satisfied

$$\theta(X, Y) = \theta(X + i, Y), \quad i = 1, 2, \ldots . \quad (14)$$

This condition is met if the undetermined coefficients A_n are periodic

$$A_n(X) = A_n(X + i) \quad i = 1, 2, \ldots \quad (15)$$

The fourth-order Runge-Kutta-Gill technique was employed. Periodic solutions were found by following the solution as X approached minus infinity. The fourth degree polynomial ($m = 4$) was found to give solutions which plotted on the same line as the $m = 6$ solutions. Comparison with the known smooth film solution (zero wave amplitude) and the pure conduction solution (neglecting axial conduction) showed complete agreement.

RESULTS

A study was made of the dependence of the heat transfer predictions upon the four model parameters: ($N_{Pe}\, h_0/4\lambda$), Z, α, and τ_s^*. Although values for these parameters are available from fluid mechanical models and experiment, these did not restrict the discussion; rather an investigation of the relative importance of the various parameters was attempted by freely varying them.

Since wave shape is important, two are investigated. One, the sine function surface, is the shape or approximately the shape assumed or predicted by the fluid mechanical models (*1, 2, 5, 7*). The other is a wave with an elongated trough and a steep crest.

For $\alpha \leq .75$, $\tau_s^* \leq +1$, $3 \geq Z \geq -3$ and $N_{Pe}\, h_0/4\lambda \leq 100$ average heat transfer for a wave length could be predicted by the conduction solution

$$N_{Nu}(\text{av.}) = \frac{1}{\sqrt{1-\alpha^2}} \quad (16)$$

with complete agreement when $N_{Pe}\, h_0/4\lambda \to 0$ and with the conduction solution being 11% low ($\tau_s^* = 1$, $Z = 3$, $\alpha = .75$) and 9% low ($\tau_s^* = 0$, $Z = 3$, $\alpha = .75$) when $N_{Pe}\, h_0/4\lambda = 100$. The Nusselt number used here is defined as the ratio of heat transferred at the wall to that which would be transferred through a smooth film of thickness h_0. This range of $N_{Pe}\, h_0/4\lambda$ values accounts for the entire region in which the fluid mechanical model might be considered to be reasonably descriptive. Kapitza (*7*) had reasoned that an appreciable increase in heat transfer would be due to the increased conduction resulting from the wave shape. This conclusion is born out by these predictions. The value of 0.75 for α is larger than the value usually encountered in the literature. It is used here to represent an extreme value of amplitude to serve as a basis for comparison. The results also show that the effect of interfacial shear is small if it does not change the wave shape, when the Nusselt number is defined as described above.

For local values of the Nusselt number at $N_{Pe}\, h_0/4\lambda = 100$, Figure 2 shows that the conduction solution gives very inaccurate predictions. However, for average Nusselt numbers at $N_{Pe}\, h_0/4\lambda = 100$ the conduction solution predicts a value within 11% of the wavy film analysis with convection. It is apparent that the difference between the local value predictions is greatly diminished when averaged over the wave length.

Experimental and theoretical values of dimensionless celerity range from 1.7 to 3.0 (*5*). Over this range there is very little effect of celerity on the average Nusselt number. Of course local or point values show a much more significant change over this celerity range. It turns out that the conduction solution is also the flowing wavy film solution for $Z = 0$. Therefore, Figure 2 also illustrates the influence of the celerity upon heat transfer ($Z = 0$ and $Z = 3$).

A wave shape that should give rise to large increases in conduction would be one with an elongated trough and sharp crest. Such a wave has been observed experimentally (*8*). A mathematical representation of this wave shape is

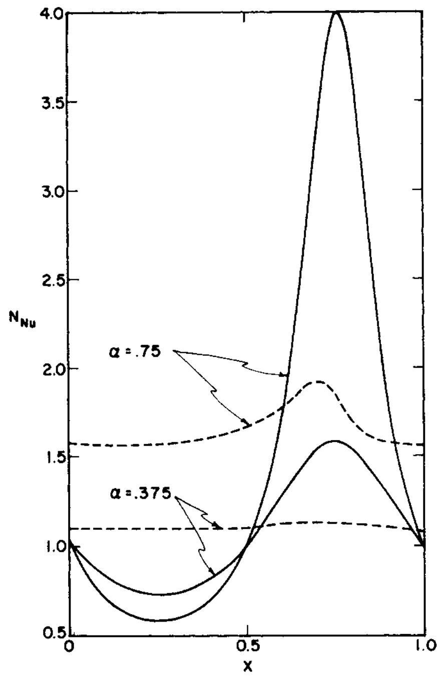

Fig. 2. Comparison of local Nusselt numbers for the conduction solution (———) with the approximate solution including both conduction and convection (- - - -). Sinusoidal wave shape, $\theta = \sum_{n=1}^{4} A_n Y^n$, $N_{Pe} h_0/4\lambda = 100$, $Z = 3$, and $\tau_s^* = 0$.

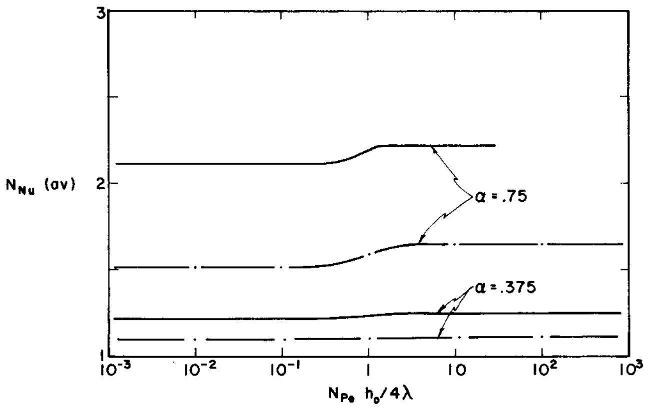

Fig. 3. A comparison of heat transfer for asymmetric and symmetric waves for $Z = 3$ and $\tau_s^* = 0$. ——— Asymmetric surface shape defined by Equation (17) where $r = 4$. — · — Symmetric sinusoidal surface shape.

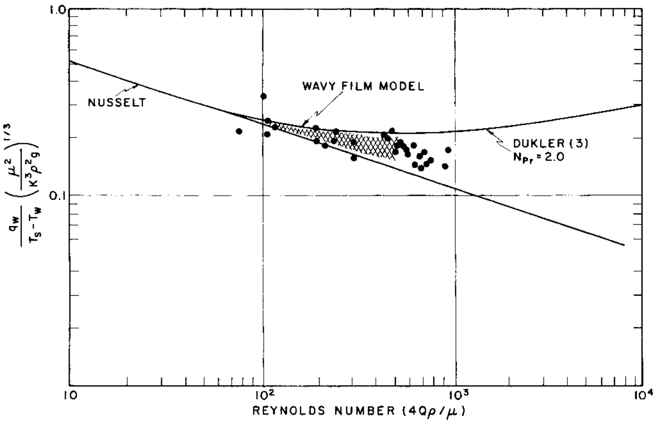

Fig. 4. Heat transfer across liquid layers. ● Data of Sikchi *(6)* for the condensation of steam *(N_{Pr} = 2)*.

given by

$$h = h_0(1 + \phi)$$

$$\phi = 2\alpha(-0.5 + \sin[\pi(1 - (X - I)^r)^2]) \quad (17)$$

where I takes on the value for 0 for $0 \leq X \leq 1$, 1.0 for $1 \leq X \leq 2$, and so forth. Results were obtained for $r = 4$ which displays a crest at $X \sim 0.75$ and troughs at 0 and 1, and $\alpha = 0.75$. These are displayed in Figure 3. It is seen that wave shape significantly influences the heat transfer bringing about enhancement over the results predicted by the sinusoidal wave shape.

To illustrate the nature of predictions based on the wavy film model, heat transfer through falling water films results are presented in Figure 4. The sine function shape wave is assumed. All physical properties are evaluated at 100°C., and the wave parameters, amplitude, wavelength and celerity, are taken from experimental data presented by Massot et al. *(5)*. The scatter of the data is represented by the shaded area on the graph. The parameter values used and model predictions are tabulated elsewhere *(4)*. Also shown in Figure 4 are data presented by Sikchi *(6)*. The wavy film model explains the observed heat transfer at least as well as the smooth laminar and turbulent film models *(3)*.

For low Reynolds numbers where wave amplitudes are near zero the model predictions correspond to the Nusselt smooth laminar film model. For higher values the wavy film model predictions fall between the Dukler turbulent film model *(3)* and the Nusselt model. No attempt is made to extend the model beyond a Reynolds number of 500. Beyond that region the wave motion is usually observed to be irregular and random, very unlike the ordered periodic motion assumed in the wavy film model.

NOTATION

c = wave celerity
C_p = specific heat
h = film thickness; surface y-coordinate
h_0 = smooth film thickness; film thickness for zero amplitude wave motion
k = thermal conductivity
N_{Nu} = Nusselt number, $\dfrac{1}{1+\phi} \cdot \dfrac{\partial \theta}{\partial Y}\Big|_{Y=0} = \dfrac{h_0 q_w}{k(T_s - T_w)}$
$N_{Nu}{}^0$ = Nusselt number for the conduction solution, $\dfrac{1}{1+\phi}$
N_{Nu} (av) = Nusselt number averaged over a wavelength
$N_{Pe}\, h_0/4\lambda$ = dimensionless group, $\dfrac{\Gamma C_p h_0}{k\lambda}$
Q = volumetric flow rate per unit breath
q_w = heat flux at the wall
t = time
T = temperature
T_s = temperature of liquid surface
T_w = temperature of the wall
u = velocity in x direction
u_0 = average velocity of smooth film of thickness h_0
$\bar{u}$ = average velocity in x direction
v = velocity in y direction
x = rectangular coordinate, directed along wall in the direction of condensate flow
x = rectangular coordinate in moving frame of reference
X = dimensionless x coordinate, $\dot{x}/\lambda$
y = rectangular coordinate, directed perpendicular to the wall
Y = dimensionless y coordinate, y/h
Z = dimensionless wave celerity, c/u_0
α = wave amplitude
Γ = condensate mass flow per unit breadth
ϕ = surface wave function
λ = wave length
μ = viscosity
θ = dimensionless temperature, $\dfrac{T - T_w}{T_s - T_w}$
ρ = density
τ_s = shear stress at the condensate surface
$\tau_s{}^*$ = dimensionless shear stress, $\dfrac{h_0 \tau_s}{u_0 \mu}$

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